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NL-2280 GE Rijswijk (NL)(54) **Self-tuning-tracking controller for permanent-magnet synchronous motors.**

(57) A self-tuning tracking controller for permanent-magnet synchronous motors is disclosed, providing for velocity or position trajectory tracking even when both the electrical and mechanical parameters of the motor, amplifier, and load are initially unknown. A time-scale simplification of a full-order mathematical model of the motor leads to a discrete-time design model that is reduced-order and that evolves in a mechanical time-scale which is substantially slower than the electrical time-scale, permitting implementation of the self-tuning tracking controller with a lower sampling frequency (and at a lower cost) than is typically expected. A piecewise-linear parameterization of the motor torque-angle characteristic functions allows for identification of higher-order harmonics with a degree of accuracy which is selectable by the user, without requiring more computation than traditional single-term sinusoidal descriptions. Excellent performance is achieved, even with very poor initial motor parameter knowledge.

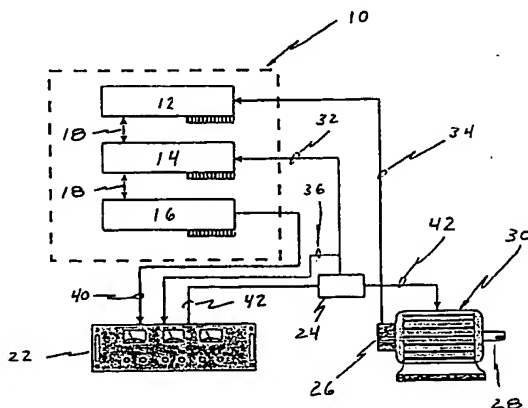


Fig. 1A

BACKGROUND OF THE INVENTION**Field of the Invention.**

- 5 This invention relates generally to adaptive control of motors and more specifically to self-tuning control of permanent-magnet synchronous motors.

Description of the related art.

- 10 Electric motors convert electrical energy into mechanical energy and come in a variety of forms and sizes depending on the specific application for which the motor is used. Electric motors use a magnetic field to form an energy link between an electrical system and a mechanical system. The magnetic field of a motor contributes to the production of mechanical output torque and induces voltages (counter emf) in coils of wire in the motor. In a permanent-magnet motor, the magnetic field is produced in part by permanent
 15 magnets mounted on a rotor (the rotating part of the motor). The stator (the stationary part of the motor) is typically wound so as to provide three sets of poles out of phase by (separated by) 120°. Permanent-magnet motors constructed in this fashion are generally referred to as synchronous if, when powered by three-phase alternating current, the motor operates in synchronism with the excitation frequency. Permanent-magnet synchronous motors are particularly appropriate for motion control applications because of
 20 their potentially very high torque-to-weight ratios, cheaper production costs, and superior thermal properties. Accordingly, permanent-magnet synchronous motors have found ready application in a wide range of environments from small computer disc drives to medium sized direct-drive robots.

- Because permanent-magnet synchronous motors are inherently nonlinear they are more difficult to control than their linear counterparts such as, for example, mechanically commutated DC motors. Only with
 25 the advent of modern nonlinear control techniques (and fast microprocessors to implement them) has this difficulty been overcome. However, a significant problem remains, in that the new control techniques normally require accurate prior knowledge of motor parameters, parameters that are either difficult to measure or change with time, or both. Generally, motion control systems include a controller, a motor, a load, and sensors. Traditional closed-loop control techniques compare a feedback signal representing the
 30 measured or sensed motor output to an input command (representing the desired motor output), then adjust the excitation applied to the motor to minimize the difference between the input command and the feedback signal. This approach works well only when possible system disturbances have been foreseen and modeled, and when the system parameters are known and remain constant over time. However, modern control techniques preferably should be able to adapt to the changing operating environment of the motor.
 35 For example, the electromagnetic characteristics of the motor may deviate substantially from nominal, bearings may become worn, the amount of friction may change, the load may vary and electronics drift may occur. When such changes to the motor and its environment occur, the control system implementing traditional control techniques can no longer provide the same accuracy initially provided and required for the particular application, despite the use of feedback. In order to correct these problems, prior art
 40 techniques generally have required manual retuning. This can be a very costly and time consuming process.

- R.B. Sepe and J.H. Lang, in their paper "Real-Time Adaptive Control of a Permanent-Magnet Synchronous Motor", IEEE Transactions on Industry Applications, Vol. 27, No. 4, pages 706-714, 1991, present a controller for permanent-magnet synchronous motors based on a simplified mathematical model
 45 of the motor in which the stator resistances, amplifier gains and parameters describing the torque-angle characteristic functions (hereinafter referred to as the "electrical parameters") are assumed to be constant and precisely known and the rotor/load inertia, cogging and other load parameters (hereinafter referred to as the "mechanical parameters") are assumed to vary slowly in an unknown fashion. The mathematical model uses an equivalent two-phase representation of the motor in which the equations are expressed in terms of
 50 the reference frame of the rotor. The goal of the controller presented in this paper is to achieve invariant velocity control in the face of varying mechanical parameters. An inner control loop comprising the motor, its inverter, its current and velocity controllers, and a state filter, is assumed to evolve in a time scale which is faster than the time scale of an outer control loop comprising a parameter estimator and a redesign algorithm for the velocity controller. The controller presented suffers from the inability of the control
 55 algorithm developed to be implemented on a standard low-cost microprocessor due to the high computational burden placed on the microprocessor by the inner loop controller. In addition, the controller presented by Sepe and Lang has limited applicability in that it is generally only applicable to permanent-magnet synchronous motors with perfectly known sinusoidal torque-angle characteristics and in which all other

electrical parameters are assumed to be known and are constant. Furthermore, the controller presented by Sepe and Lang is limited to velocity control. Thus, a need yet exists for a controller for permanent-magnet synchronous motors which is applicable to permanent-magnet synchronous motors regardless of their torque-angle characteristics and in which all the electrical and mechanical parameters of the motor are unknown or vary over time. Moreover, a need still exists for a self-tuning controller for permanent-magnet synchronous motors which may be implemented with a standard, low-cost microprocessor without placing an excessive computational burden thereon.

In a second paper by Sepe and Lang, "Real-Time Observer-Based (Adaptive) Control of a Permanent-Magnet Synchronous Motor without Mechanical Sensors", IEEE Transactions on Industry Applications, Vol. 28, No. 6, pages 1345-1352, 1992, an adaptive velocity controller is presented based on a mechanically sensorless, full-state observer which is applied to a mathematical model of the motor identical to the one presented in their above-described publication. The controller presented in this second paper also assumes that the torque-angle characteristics of the motor are sinusoidal and known and that the electrical parameters of the motor are known and remain constant, while the mechanical parameters of the motor are permitted to vary slowly. Although a discrete-time estimation of the mechanical parameters of the motor is employed, the controller suffers from many of the shortcomings identified above with respect to their previously-referenced work.

Accordingly, there is yet a need for a computationally efficient controller which is capable of changing control instructions (i.e. self-tuning) in accordance with specified performance criteria when any or all of the system parameters are unknown or change with time. It is to the provision of such a controller and technique that the present invention is primarily directed.

SUMMARY

The present invention is directed to an inexpensive self-tuning controller for permanent-magnet synchronous motors. The method according to the invention comprises the steps of applying a voltage to the stator windings so as to command the motor to follow a desired position or velocity trajectory using initial estimates of the electrical and mechanical parameters of the motor in a simplified mathematical model of the motor including piecewise-linear (or piecewise-polynomial) approximations for the torque-angle characteristic functions of the motor and, optionally, a piecewise approximation for the motor load. After selecting the initial estimates of the electrical and mechanical parameters of the motor, voltage is applied to the stator windings, and rotor position, velocity, and currents in the stator windings are measured. The measured rotor position and velocity are compared to a specified (desired) velocity or position trajectory for the motor and error signals for position and velocity are obtained. These error signals are used to develop updated parameter estimates, yielding an updated simplified mathematical model of the motor. Self-tuning control is obtained by subsequently applying a voltage to the stator windings so as to command the motor to follow the specified or desired position or velocity trajectory using the updated simplified mathematical model of the motor. The simplified model of the motor is a discrete-time, reduced-order model.

The new piecewise-linear parameterization of the motor torque-angle characteristic functions allows for identification of higher order harmonics with a degree of accuracy which is selectable by the end user of the controller without requiring more computations than with known one-term sinusoidal representations (e.g., a one-term Fourier series) of the motor torque-angle characteristic functions. Excellent motor performance thereby is achieved due to computationally efficient self-tuning, even when the electrical and mechanical parameters of the motor are unknown prior to beginning operation.

Thus, it is an object of the present invention to provide an automatic method of controlling permanent-magnet synchronous motors wherein parameters associated with the motor and its load are continuously estimated so as to maintain optimum motor control.

It is another object of this invention to provide a self-tuning controller for permanent-magnet synchronous motors which is inexpensive to manufacture.

It is a further object of the present invention to provide a self-tuning controller for permanent-magnet synchronous motors which may be implemented using a low-cost microprocessor.

It is another object of this invention to provide a self-tuning controller for permanent-magnet synchronous motors which achieves robust performance even when both the electrical and mechanical parameters of the motor, including its torque-angle characteristics, initially are unknown and/or change over time.

A further object of the present invention is to provide a method of controlling permanent-magnet synchronous motors which experiences minimal error in following a specified position or velocity trajectory.

These and other objects, features, and advantages of the present invention will become apparent upon reading the following specification in conjunction with the accompanying drawing figures.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1A is a graphical representation of a motor control arrangement which has been used to demonstrate the present invention.

5 Fig. 1B is a graphical representation of a preferred, low-cost implementation of the present invention.

Fig. 2A is a schematic block diagram of primary steps of the control scheme according to the present invention.

Fig. 2B is a flow diagram illustrating the program implemented by the self-tuning method of the present invention.

10 Fig. 3 is an illustration of a piecewise-linear approximation of a sinusoidal torque-angle characteristic function.

Fig. 4 is an illustration of shape functions which can be used to describe, mathematically, the function shown in Fig. 3.

Fig. 5 is a block diagram representation of a control system according to the present invention.

15 Fig. 6 is a waveform diagram illustrating measured torque-angle characteristic functions and cogging for a permanent-magnet synchronous motor.

Fig. 7 is a waveform diagram illustrating initial (poorly) estimated torque-angle characteristic functions for the motor of Fig. 6.

20 Fig. 8A is a graphical illustration of self-tuning velocity trajectory tracking control of the permanent-magnet synchronous motor of Fig. 6 using the present invention.

Fig. 8B is a graphical illustration of tracking error attained by the present invention in following the velocity trajectory of Fig. 8A.

Fig. 8C is a graphical illustration of the instantaneous power supplied to the motor of Fig. 6 in following the velocity trajectory of Fig. 8A.

25 Fig. 9A is a graphical illustration of self-tuning position trajectory tracking control of the permanent-magnet synchronous motor of Fig. 6 using the present invention.

Fig. 9B is a graphical illustration of tracking error attained by the present invention in following the position trajectory of Fig. 9A.

30 Fig. 9C is a graphical illustration of the instantaneous power supplied to the motor of Fig. 6 in following the position trajectory of Fig. 9A.

DETAILED DESCRIPTION OF THE INVENTION

Referring now to the drawings, wherein like numerals denote like parts throughout the several views, 35 Fig. 1A illustrates a prototype apparatus which has been constructed to demonstrate the efficacy of the self-tuning method of the present invention. In reading the following description of the prototype apparatus actually constructed and tested, it should be borne in mind that many modifications can be made therein, such as replacing one commercially available electronic chip with another.

I. The Prototype Apparatus

40 The arrangement of Fig. 1A includes a computer 10 for controlling motor 30 and its load at shaft 28 via amplifier 22, based on the rotor position and velocity as sensed by encoder 26. According to the prototype device actually constructed, computer 10 is an Intel 80486-based personal computer with a floating point 45 digital signal processor (DSP) card 12, part number 600-01011 from Spectrum Signal Processing, Inc. of Vancouver British Columbia, Canada. The self-tuning method of the present invention is implemented by the 32 bit floating point DSP processor board 12 which is connected to analog input board 14 and the 1000 line encoder 26. Analog input board 14 is a 32-channel board from Spectrum Signal Processing, Inc., part number 600-00257. Analog input board 14 is connected to processor board 12 via DSP link 18, a high- 50 speed parallel bus. Communication over this bus proceeds independent of the CPU of host computer 10, which allows computer 10 to be dedicated to other functions such as the plotting of results. The processor board 12 is also connected to analog output board 16 via the DSP link 18. Analog output board 16 is a 16 channel board from Spectrum Signal Processing, Inc., part number 600-00428.

Also shown in Fig. 1A is permanent-magnet synchronous motor 30 which may be used to move a load 55 connected to shaft 28 in accordance with a desired position or velocity trajectory. Encoder 26 is attached to the back of motor 30 so that the position and velocity of the rotor of motor 30 (connected to shaft 28) may be determined by conventional techniques and supplied to processor board 12 over line 34. Power is supplied to motor 30 by a three-phase, power op-amp based linear amplifier 22, with the stator currents of

motor 30 being obtained by measuring a voltage drop across current sensors 24, which are generic 1 ohm, 20 watt power resistors connected in series with the stator windings of motor 30. Amplifier 22 drives motor 30 over line 42 based on command signals received from analog output board 16 over line 40 and current feedback signals received from current sensors 24 over line 36.

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II. A Preferred Apparatus

Fig. 1B illustrates a preferred, low-cost implementation for the self-tuning method of the present invention. This arrangement includes a low-cost controller board 43 controlling the motor 30 and its load at shaft 28 via a low-cost pulse-width modulation (PWM) amplifier 44, based on the rotor position and velocity measured with encoder 26. The controller board 43 is based on a low-cost, fixed point 16 bit microprocessor, such as the MC68HC16 microcontroller from Motorola. Controller board 43 also includes encoder interface circuitry for converting the quadrature encoder signals of line 48, analog to digital conversion circuitry for measurement of the stator current signals of line 47, and digital to analog conversion circuitry for commanding the amplifier 44 over line 45. Based on stator current measurements from current sensors 24 and rotor position and velocity measurements derived from encoder 26, the microprocessor of controller board 43 applies excitation to the motor 30 via the PWM amplifier 44 using lines 45 and 46.

The amplifier 44 is a low-cost, 3 phase PWM type switching amplifier, based on either a standard inverter configuration or a unipolar H-bridge configuration. Based on input from the controller board 43 via line 45, and measurements of the stator currents from the current sensors 24 via line 47, the amplifier 44 commands the stator voltages of the motor 30 via line 46.

As in Fig. 1A, the motor 30 is a permanent-magnet synchronous motor, which drives the load via shaft 28. Attached to shaft 28 is also encoder 26, which provides for quadrature signals 48 which may be used to measure the position and velocity of shaft 28. The current sensors 24 may be generic power resistors, connected in series with the stator windings of the motor 30, or they may be Hall-effect current sensors, or they may be SenseFet-based current sensors.

III. The Method Carried Out By the Control Scheme

Fig. 2A shows an overview of the control scheme according to the present invention. Block 51 depicts the initial step of initializing the system wherein the encoder 26 is initialized as well as the parameter estimates for the motor. These initial estimates of the motor's characteristics needn't be particularly accurate because of the self-tuning (self-correcting) nature of the invention. These initial estimates of the motor's characteristics are used in a mathematical model, described in more detail below, to calculate an initial excitation to be applied to the motor to urge the motor toward a desired position and/or velocity trajectory. The initial excitation is then applied to the motor, as per block 52. The response of the motor to the initial excitation is detected by determining the new rotor position, rotor velocity, and the stator currents (block 53). This information about how the motor performed in response to the excitation is then used to calculate an updated model of the motor (block 54). As depicted by block 55, the updated motor model is used, along with the rotor position and rotor velocity information, to calculate a new excitation to be applied to the motor. According to block 56 this new excitation is then applied to the motor and the cycle of observing the motor's performance, updating the motor model, calculating a new excitation, and applying the new excitation repeats over and over (blocks 53 - 56).

The preferred embodiment of the self-tuning method of the present invention is illustrated in more detail in Fig. 2B. The method begins in step 60 with the initialization of the unknown parameters to some nominal value such as those which may be supplied by manufacturers' data sheets. Step 60 also includes the initialization of encoder 26 either using a hardware zero reference or by performing an initialization sequence on the motor. It should be noted that the performance of an initialization sequence on the motor is not required if an absolute position sensor is used as encoder 26. After the initialization is complete, the repetitive part of the self-tuning method of the present invention (i.e., steps 62, 64, 66, 68, 70, 72, and 74) is entered. The first step in this loop involves measuring rotor position, $\theta[n]$, via encoder 26 and measuring the stator currents, $i[n]$, via current sensors 24. Step 62 may be carried out by any conventional measuring techniques with any necessary conversion so that appropriate measurement units are obtained. Next, in step 64, rotor velocity, $\omega[n]$, is computed using, for example, any appropriate numerical differentiator operating on the measured $\theta[n]$ from step 62. This step typically includes using a low pass filter which attenuates any noise which may result from the numerical differentiation. In step 66, the known last input $u[n-1]$ the measured $i[n]$ and $\theta[n]$ from step 62, and the computed $\omega[n]$ from step 64 are used to compute updated electrical parameter estimates, $\hat{\theta}_e[n+1]$.

One of the electrical parameter estimates computed in step 66 is the torque-angle characteristic function. A new piecewise-linear (or piecewise-polynomial) approximation of the torque-angle characteristic function of motor 30 is used in accordance with the teachings of the present invention as is more fully discussed below. In addition, a piecewise approximation for the motor load may be used. In step 68, updated mechanical parameter estimates, $\hat{\theta}_m[n+1]$, are computed using the known last input, $u[n-1]$, the measured rotor position, $\theta[n-1]$, the computed rotor velocities, $\omega[n]$ and $\omega[n-1]$, and the estimated electrical parameters, $\hat{\theta}_e[n]$. Next, in step 70, the measured position, $\theta[n]$, the computed velocity, $\omega[n]$, and the estimated electrical and mechanical parameters, $\hat{\theta}_e[n+1]$ and $\hat{\theta}_m[n+1]$, respectively, as well as the desired rotor position or velocity, $\theta_d[n]$ or $\omega_d[n]$, respectively, are used to compute the new control input, $u[n]$. Control is implemented using an error-driven normalized gradient parameter update law based on a discrete-time, reduced-order mathematical model of a permanent-magnet synchronous motor which evolves in the mechanical time-scale which is substantially slower than the electrical time-scale of the motor. This feature of the present invention is also discussed in further detail below. Then, in step 72, the new control input, $u[n]$, is applied to motor 30 via a digital to analog converter in controller board 43 and amplifier 44. Finally, in step 74, the index variable, n , is increased by 1 and the self-tuning method of the present invention waits until the next sampling instant, $t=nT$ where T is the sampling period, before steps 62 through 74 are repeated.

The method of the present invention, thus described, has been carried out and verified using the arrangement of Fig. 1A and more generally, preferably is implemented using the preferred low-cost apparatus of Fig. 1B. Having now described the method of the present invention in its preferred form, what follows is the mathematical basis for the method of the present invention.

In modeling the permanent-magnet synchronous motor, it is assumed that the motor is magnetically linear with a smooth air gap and that hysteresis is negligible. Thus, the design begins with the following full-order mathematical model

$$\frac{d\theta}{dt} = \omega \quad (1)$$

$$\frac{d\omega}{dt} = \frac{1}{J} (-\tau_L(\theta, \omega, t) + i'K(\theta)) \quad (2)$$

$$L \frac{di}{dt} = -Ri - \omega K(\theta) + v \quad (3)$$

where θ and ω respectively are the angular rotor position and velocity, i is an M vector of stator phase currents, J is the rotor moment of inertia, $\tau_L(\theta, \omega, t)$ is the load torque, $K(\theta)$ is a vector of torque-angle characteristic functions, L is a diagonal matrix of stator phase self-inductances, R is a diagonal matrix of stator phase resistances, v is a vector of phase input voltages, and the $'$ denotes algebraic transposition. It is further assumed that the motor has N_p magnetic pole pairs on the rotor, implying that the torque-angle characteristic function is periodic according to the following equation

$$K(\theta) = K(\theta + 2\pi/N_p) \quad (4)$$

Important simplifications to this model may be made if the electrical dynamics are significantly faster than the mechanical dynamics. In order to enhance the speed of the electrical dynamics, an inner-loop analog current feedback is employed. The feedback signal is given by

$$v := -K_{amp}i + u \quad (5)$$

where K_{amp} is a diagonal gain matrix and u is an M vector of digital inputs

$$u(t) = u(nT) =: u[n], \forall t \in [nT, (n+1)T], n = 0, 1, \dots \quad (6)$$

The sampling period T , which is under the designer's control, is assumed to be chosen on the basis of the *mechanical* dynamics (i.e., at about 1ms, T is larger than it would normally be if it were chosen with respect to the faster electrical dynamics). Under this inner-loop feedback, the electrical dynamics as described in Equation 3 are rewritten as

$$L \frac{di}{dt} = -R_e i - \omega K(\theta) + u[n] \quad (7)$$

5

where

$$R_e := R + K_{amp} \quad (8)$$

10

is the effective resistance.

Many commercially available permanent-magnet synchronous motors have phase inductances L that are negligibly small. In this case, the reduced-order discrete-time design model

$$\theta[n+1] = \theta[n] + T\omega[n] + \nu_1[n] \quad (9)$$

15

$$\begin{aligned} \omega[n+1] = \omega[n] + \frac{T}{J} & \left(-\tau_L(\theta[n], \omega[n], nT) \right. \\ & \left. - \omega[n] K'(\theta[n]) R_e^{-1} K(\theta[n]) + u'[n] R_e^{-1} K(\theta[n]) \right) + \nu_2[n] \end{aligned} \quad (10)$$

20

is used, where $\theta[n] := \theta(nT)$, $\omega[n] := \omega(nT)$ and where $\nu_1[n]$ and $\nu_2[n]$ are disturbance terms. The electrical variable algebraic constraint

25

$$R_e i[n] + \omega[n] K(\theta[n]) = u[n-1] + \nu_3[n] \quad (11)$$

30

also is used, where $i[n] := i(nT)$ and $\nu_3[n]$ is a disturbance term. The delay in the input $u[n-1]$ in this equation is a residual effect of the neglected fast dynamics. K. R. Shouse and D. G. Taylor, in the paper entitled "Observer-Based Control of Permanent-Magnet Synchronous Motors," published in the *Proceedings of the 1992 International Conference on Industrial Electronics, Control, Instrumentation and Automation*, pp. 1482-1487 (November 1992), the content of which is incorporated herein by reference, show that if the analog feedback gain K_{amp} is on the order of 10 or less, and if the mechanical states are bounded, then the disturbances $\nu_1[n]$, $\nu_2[n]$ and $\nu_3[n]$ are $O(L + T^2)$, meaning that they asymptotically go to zero as L and T go to zero. Thus, if L and T are sufficiently small (and if K_{amp} is about 10 or less and the mechanical states are bounded), then the disturbance terms $\nu_1[n]$, $\nu_2[n]$ and $\nu_3[n]$ may be neglected.

35

If the phase inductances L are not particularly small, then the technique described in the above noted paper of Shouse and Taylor may be extended by choosing the modified analog feedback

40

$$v := K_{amp}(u[n] - i) \quad (12)$$

and by choosing K_{amp} to be large, thereby causing the amplifier to operate in a current-tracking mode. In this case, the inverse of the analog gain K_{amp} is taken as negligible, and the reduced-order discrete-time design model

45

$$\theta[n+1] = \theta[n] + T\omega[n] + \nu_1[n] \quad (13)$$

$$\omega[n+1] = \omega[n] + \frac{T}{J} (-\tau_L(\theta[n], \omega[n], nT) + u'[n] K(\theta[n])) + \nu_2[n] \quad (14)$$

50

would be used, along with the associated electrical variable algebraic constraint

$$R u[n-1] + \omega[n] K(\theta[n]) = v(nT^-) + \nu_3[n] \quad (15)$$

55

where $v(nT^-)$ is the amplifier output voltage measured just prior to the application of $u[n]$. By proper extension of the results in the above noted paper of Shouse and Taylor, it may be shown that if the mechanical states are bounded, then the disturbances $\nu_1[n]$, $\nu_2[n]$ and $\nu_3[n]$ are $O(T^2 + 1/K_{amp})$, implying

that for sufficiently large K_{amp} and small T , the disturbances $v_1[n]$, $v_2[n]$ and $v_3[n]$ may be neglected.

In the remainder of the development, the reduced-order discrete-time model of Equations 9-11 will be used, thereby implying the assumption of sufficiently small stator inductances L . The extension of the following material to motors with non-negligible inductances L , by using the alternate model of Equations 13-15 will be clear to those skilled in the art.

Prior to parameterizing the system, it is first necessary to approximate $K(\theta)$ with a function that depends on only a finite number of fixed (with respect to θ) parameters. A known prior method for accomplishing this is to approximate $K(\theta)$ with a truncated Fourier series, with the parameters being the Fourier coefficients. This method, however, suffers from several disadvantages. First, some torque-angle characteristics require many terms from the Fourier series for an accurate approximation, leading to a large number of unknown parameters and a high parameter update computational burden. Furthermore, Fourier expansions require transcendental function evaluations, which require significant computation.

One of the main contributions of the present invention is a new structural approximation of $K(\theta)$. Specifically, a *piecewise-linear* approximation of $K(\theta)$ is used, an example of which is shown in Fig. 3. Alternatively, $K(\theta)$ can be described piecewise by polynomials. In the piecewise-linear formulation, the electrical period $\theta_p := 2\pi/N_p$ is divided into N_s intervals, and $K(\theta)$ is approximated with an affine function over each interval. Because of the periodicity (see Equation 4), the approximation of $K(\theta)$ over one electrical period serves as an approximation over all θ . It should be clear that any periodic function (with a bounded first derivative) may be approximated to any desired degree of accuracy using the piecewise-linear approximation by simply choosing N_s to be large enough. Note that the intervals over which the function is assumed to be affine need not all be of the same length. For simplicity, however, equal length intervals are chosen as shown in Fig. 3.

To formalize this piecewise-linear parameterization, the *shape functions*

$$s_j(\theta) := \begin{cases} \frac{|\theta| - (j-1)\Delta\theta}{\Delta\theta}, & |\theta| \in [(j-1)\Delta\theta, [(j-1)\Delta\theta] + \Delta\theta) \\ \frac{(j+1)\Delta\theta - |\theta|}{\Delta\theta}, & |\theta| \in [j\Delta\theta, (j+1)\Delta\theta) \\ 0, & \text{else} \end{cases} \quad (16)$$

are defined for $j = 0, \dots, N_s - 1$, where

$$[\cdot] := \text{mod}(\cdot, \theta_p)$$

and where $\Delta\theta := \theta_p/N_s$. Although the definition in Equation 16 appears computationally complex, the graphical description in Fig. 4 shows that the functions are conceptually simple. It should be further noted that evaluation of a shape function requires only one modulo and one multiply operation (despite the definition in Equation 16, and in contrast to a $\sin(\cdot)$ call).

With these shape functions, the piecewise-linear approximation of the j th element of $K(\theta)$ can be written as the linear-in-parameter description

$$\tilde{K}_j(\theta) := S'(\theta)\Theta_{K_j}^* \quad (17)$$

where

$$S(\theta) := \begin{bmatrix} s_0(\theta) & \cdots & s_{N_s-1}(\theta) \end{bmatrix}' \quad (18)$$

$$\Theta_{K_j}^* := \begin{bmatrix} K_j(0) \\ \vdots \\ K_j(\Delta\theta(N_s - 1)) \end{bmatrix} \quad (19)$$

Note that the shape functions basically provide a means of writing $K(\theta)$ over each interval as a convex combination of the interval endpoint values.

As shown in Fig. 3, the piecewise-linear parameterization of Equation 17 may require more parameters than a Fourier truncation approximation of similar accuracy. In this case, why would one choose the piecewise-linear formulation over the truncated Fourier series? Besides the aforementioned advantage of not requiring transcendental function calls, there is another significant advantage to the piecewise-linear parameterization which is not readily evident from Equations 17-19. Using the definitions

$$\theta_l := \text{floor} \left(\frac{\lfloor \theta \rfloor}{\Delta \theta} \right) \quad (20)$$

$$\theta_u := \text{mod} \left(\text{floor} \left(\frac{\lfloor \theta \rfloor}{\Delta \theta} \right) + 1, N_s \right) \quad (21)$$

it is seen from Equation 16 (and from Fig. 4) that

$$j \notin \{\theta_l, \theta_u\} \Rightarrow s_j(\theta) = 0 \quad (22)$$

But this means that Equation 17 can be greatly simplified. Taking Equation 22 into account, the piecewise-linear approximation $\tilde{K}_j(\theta)$ may be more simply written as

$$\tilde{K}_j(\theta) = \tilde{S}'(\theta) \tilde{\Theta}_{K_j} \quad (23)$$

where

$$\tilde{S}(\theta) := \begin{bmatrix} s_{\theta_l}(\theta) & s_{\theta_u}(\theta) \end{bmatrix}' \quad (24)$$

$$\tilde{\Theta}_{K_j} := \begin{bmatrix} K_j(\theta_l \Delta \theta) & K_j(\theta_u \Delta \theta) \end{bmatrix}' \quad (25)$$

The simplified formulation of Equation 23 reveals that evaluation of the approximate $\tilde{K}_j(\theta)$ requires only 4 multiplies and 2 modulus. It should be noted, however, that the parameter vector

$$\tilde{\Theta}_{K_j}$$

is not complete, in the sense that it does not contain all of the parameters necessary to approximate $K_j(\theta)$ for all values of θ . Finally note that because it requires more parameters, the piecewise-linear parameterization will usually require more computer memory than a comparably accurate truncated Fourier series (at least for functions with small higher order harmonics). However, one would expect that accurate piecewise-linear parameterizations would require at most a few hundred parameters, and the memory costs under this assumption are inconsequential.

With the piecewise-linear approximate formulation of $\tilde{K}(\theta)$ complete, the design model of Equations 9-11 may now be written as a linear expression of the unknown parameters. It is assumed that the parameters of the piecewise-linear approximation $\tilde{K}(\theta)$, the effective resistances R_e , the rotor inertia J and any parameters associated with the load torque $\tau_L(\theta[n], \omega[n], nT)$ are all unknown.

Using the parameterization of Equation 23, the electrical variable expression in Equation 11 is rearranged to obtain the linear-in-parameter *inner-loop output equation*

$$y_e[n] := u_i[n-1] \quad (26)$$

$$= w'_{e_j}[n] \Theta_{e_j} + O(L + T^2) \quad (27)$$

for $j = 1, \dots, M$, where

$$w_{e_j}[n] := \begin{bmatrix} i_j[n] & \omega[n] \tilde{S}'(\theta[n]) \end{bmatrix}' \quad (28)$$

$$\Theta_{e_j}^* := \begin{bmatrix} R_{e_j} & (\tilde{\Theta}_{K_j}^*)' \end{bmatrix}' \quad (29)$$

To formulate an output equation containing the mechanical parameters, it is first assumed that the load torque can be linearly parameterized as

$$\tau_L(\theta[n], \omega[n], nT) = w_r'[n] \Theta_r^* \quad (30)$$

where the regressor $w_r[n]$ is a function of only the known quantities $\theta[n]$, $\omega[n]$ and nT . With this parameterization, the linear-in-parameter *outer-loop output equation* is written as

$$y_m(\Theta_e^*, n-1) := \omega[n-1] \tilde{K}'(\theta[n-1]) R_e^{-1} \tilde{K}(\theta[n-1]) - u'[n-1] R_e^{-1} \tilde{K}(\theta[n-1]) \quad (31)$$

$$= w_m'[n] \Theta_m^* + O(L + T^2) \quad (32)$$

where

$$\Theta_e^* := \begin{bmatrix} \Theta_{e_1}^* & \dots & \Theta_{e_M}^* \end{bmatrix} \quad (33)$$

$$w_m[n] := \begin{bmatrix} \frac{\omega[n-1] - \omega[n]}{T} \\ -w_r[n-1] \end{bmatrix} \quad (34)$$

$$\Theta_m^* := \begin{bmatrix} J \\ \Theta_r^* \end{bmatrix} \quad (35)$$

The dependence of y_m on the electrical parameters Θ_e^* will require a nested identifier structure.

The self-tuning controller may be formulated on the basis of Equations 9-11, 27-29 and 32-35. Fig. 5 depicts the permanent-magnet synchronous motor self-tuning controller of the present invention in block diagram form. Blocks 80, 82, and 84, marked F_m , F_a and F_r , respectively, constitute the digital controller. Blocks 86 and 88, marked "Inner-Loop Identifier" and "Outer-Loop Identifier," respectively, make up the parameter identifiers, while block 90, marked K_{amp} , is the analog current feedback loop associated with the power amplifier (not shown) which drives motor 30. The following describes these components in more detail:

The "Inner Loop Identifier" block 86 takes the current measurement $i[n]$, the amplifier input $u[n-1]$, and the rotor position and velocity $\theta[n]$ and $\omega[n]$ and computes the next electrical parameter estimate $\hat{\Theta}_e[n+1]$ per Equation 47, listed below.

The "Outer Loop Identifier" block 88 takes the amplifier input $u[n-1]$, the rotor position and velocity $\theta[n]$ and $\omega[n]$, and the electrical parameter estimate $\hat{\Theta}_e[n+1]$ and computes the next mechanical parameter estimate $\hat{\Theta}_m[n+1]$ per Equation 49, listed below.

The F_m block 80 takes the rotor position and velocity $\theta[n]$ and $\omega[n]$, and the desired position $\theta_d[n]$ (for position control) or desired velocity $\omega_d[n]$ (for velocity control), and computes the desired acceleration signal $\alpha_d[n]$ per Equations 43-44, listed below.

The F_a block 82 takes the desired acceleration $\alpha_d[n]$, the rotor position and velocity $\theta[n]$ and $\omega[n]$ and the mechanical parameter estimate $\hat{\Theta}_m[n+1]$ and computes the desired torque $\tau_d[n]$ per Equations 39-40, listed below (where Θ_m^* is replaced by $\hat{\Theta}_m[n+1]$).

The F_r block 84 takes the desired torque $\tau_d[n]$, the rotor position and velocity $\theta[n]$ and $\omega[n]$ and the

electrical parameter estimate $\hat{\Theta}_e[n + 1]$, and computes the amplifier input command $u[n]$ per Equations 36-37, listed below (where Θ_e^* is replaced by $\hat{\Theta}_e[n + 1]$).

The K_{amp} block 90, which is internal to the power amplifier, takes the analog current measurement $i(t)$ and the amplifier input $u[n]$, and outputs the voltage $u[n] - K_{amp}i(t)$.

5 The first step in the self-tuning tracking controller formulation is the construction of a torque/acceleration linearizing control. Assuming for the moment that Θ_e^* is known and defining for convenience $x[n] := [\theta[n], \omega[n]]'$, the motor input is commanded according to

$$10 \quad u[n] = F_r(x[n], \tau_d[n], \Theta_e^*) \quad (36)$$

where

$$15 \quad F_r(x[n], \tau_d[n], \Theta_e^*) := \omega[n] \bar{K}(\theta[n]) + R_e \frac{\bar{K}(\theta[n])}{\|\bar{K}(\theta[n])\|^2} \tau_d[n] \quad (37)$$

20 and $\tau_d[n]$ is a desired torque signal. Under this control, the rotor velocity dynamics satisfy

$$25 \quad \omega[n + 1] = \omega[n] + \frac{T}{J} \left(-\tau_L(\theta[n], \omega[n], nT) + \tau_d[n] + O(L + T^2) \right) \quad (38)$$

Assuming for the moment that Θ_m^* is known, the desired torque is chosen as

$$30 \quad \tau_d[n] = F_o(x[n], \alpha_d[n], \Theta_m^*) \quad (39)$$

where

$$35 \quad F_o(x[n], \alpha_d[n], \Theta_m^*) := \tau_L(\theta[n], \omega[n], nT) + J\alpha_d[n] \quad (40)$$

40 and $\alpha_d[n]$ is a desired acceleration signal. It is easy to see that with $\tau_d[n]$ chosen according to Equation 39, the motor mechanical dynamics satisfy

$$45 \quad \begin{bmatrix} \theta[n + 1] \\ \omega[n + 1] \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta[n] \\ \omega[n] \end{bmatrix} + \begin{bmatrix} 0 \\ T \end{bmatrix} \alpha_d[n] + O(L + T^2) \quad (41)$$

(which is a disturbance away from a linear controllable system).

Of course, the unknown parameter vectors Θ_e^* and Θ_m^* are not available, so the implementable control law

$$50 \quad u[n] = F_r(x[n], F_o(x[n], \alpha_d[n], \hat{\Theta}_m[n]), \hat{\Theta}_e[n]) \quad (42)$$

is used, where $\hat{\Theta}_e[n]$ and $\hat{\Theta}_m[n]$ are parameter estimates which are supplied by the identifiers to follow.

55 With the linearizing torque/acceleration control formulation complete, the motion tracking controller that determines the $\alpha_d[n]$ necessary to achieve either velocity or position trajectory tracking may now be formulated. Toward this end, the desired acceleration is chosen as

$$\alpha_d[n] = F_m(x[n], x_d[n]) \quad (43)$$

where $x_d[n] := [\theta_d[n], \omega_d[n]]'$ is a vector of desired rotor position and velocity at $t = nT$. The function $F_m(\cdot, \cdot)$ is defined by

$$F_m(x[n], x_d[n]) := \begin{cases} \frac{1}{T}(K_\omega \epsilon_\omega[n] - \omega[n] + \omega_d[n+1]), & \text{velocity control} \\ \frac{1}{T}(K_\theta \epsilon_\theta[n] + K_\omega \epsilon_\omega[n] - \omega[n] + \omega_d[n+1]), & \text{position control} \end{cases} \quad (44)$$

where $\epsilon_\omega[n] := \omega[n] - \omega_d[n]$ and $\epsilon_\theta[n] := \theta[n] - \theta_d[n]$ and where K_θ and K_ω are design gains.

Under velocity tracking control, the choice of $\alpha_d[n]$ given by Equation 43 results in velocity error dynamics which satisfy

$$\epsilon_\omega[n+1] = K_\omega \epsilon_\omega[n] + O(L + T^2) \quad (45)$$

Clearly, if $|K_\omega| < 1$ and $\omega_d[n]$ is appropriately bounded, and if

$$\hat{\Theta}_e[n] = \Theta_e^* \text{ and } \hat{\Theta}_m[n] = \Theta_m^*$$

for all $n \geq 0$, then the control yields tracking error which exponentially decays to an $O(L + T^2)$ neighborhood of zero.

Under position tracking control, the position trajectory may be arbitrarily specified. The "desired" velocity is chosen, however, not arbitrarily, but according to the rule $\omega_d[n] := (\theta_d[n+1] - \theta_d[n])/T$. Under this restriction, the choice of $\alpha_d[n]$ given by Equation 43 gives mechanical dynamics

$$\begin{bmatrix} \epsilon_\theta[n+1] \\ \epsilon_\omega[n+1] \end{bmatrix} = \begin{bmatrix} 1 & T \\ K_\theta & K_\omega \end{bmatrix} \begin{bmatrix} \epsilon_\theta[n] \\ \epsilon_\omega[n] \end{bmatrix} + O(L + T^2) \quad (46)$$

Choosing $K_\theta < 0$ and $\frac{1}{2}K_\theta - 1 < K_\omega < TK_\theta + 1$, if $\theta_d[n]$ is appropriately bounded and if

$$\hat{\Theta}_e[n] = \Theta_e^* \text{ and } \hat{\Theta}_m[n] = \Theta_m^*$$

for $n \geq 0$, then the position tracking error decays to an $O(L + T^2)$ neighborhood of zero.

Since the parameters are not known precisely, it is necessary to design adaptive update laws which will identify them. Using a robust normalized gradient update law (known to those skilled in the art) for the linear-in-parameter inner-loop output equation, Equation 27, gives the electrical parameter estimate update law

$$\hat{\Theta}_{e,j}[n+1] = \hat{\Theta}_{e,j}[n] + \frac{\gamma_e w_{e,j}[n]}{\kappa_e + w'_{e,j}[n] w_{e,j}[n]} G_{\beta_e}(y_{e,j}[n] - w'_{e,j}[n] \hat{\Theta}_{e,j}[n]) \quad (47)$$

for $j = 1, \dots, M$, where γ_e is a diagonal matrix of design gains, κ_e and β_e are design parameters and

$$G_{\beta}(x) := \begin{cases} x, & |x| \geq \beta \\ 0, & |x| < \beta \end{cases} \quad (48)$$

is a dead-zone function which improves robustness.

Because of the simplification resulting from Equation 22, the update of Equation 47 requires only about 15 flops/phase, regardless of the number of segments N_s . This means that the piecewise-linear

parameterization requires less computation than a two parameter (magnitude and phase) Fourier truncation. Thus, for any torque-angle characteristic which has even a single harmonic, the new technique *consistently* gives greater accuracy than the truncated Fourier series of comparable update computational complexity.

Using the robust normalized gradient update law with the outer-loop output equation, Equation 32, gives the mechanical parameter estimate update law

$$\hat{\Theta}_m[n+1] = \hat{\Theta}_m[n] + \frac{\gamma_m w_m[n]}{\kappa_m + w'_m[n] w_m[n]} G_{\beta_m}(y_m(\hat{\Theta}_m[n], n-1) - w'_m[n] \hat{\Theta}_m[n]) \quad (49)$$

where γ_m is a diagonal matrix of design gains, κ_m and β_m are design parameters and the dead-zone on the error term is again used for improved robustness.

For the purpose of demonstrating the present invention, a laboratory prototype system has been constructed. All computer code associated with control and data acquisition for the laboratory prototype was implemented in the programming language "C" (one of the primary advantages of the setup). The sampling period achievable for the self-tuning tracking controller of the present invention using a 32-bit floating point digital signal processor is on the order of 0.6 msec, and all results which follow, demonstrating the operation of the present invention, use a sampling period of $T = 1$ msec. If the computer code were implemented in faster assembly language (using fixed point math), then this sampling period could be attained using an inexpensive microprocessor.

Referring again to Fig. 1A, the load torque of the motor 30 is the sum of viscous friction and magnetic cogging

$$\tau_L(\theta, \omega, t) = B\omega + \eta(\theta) \quad (50)$$

The nominal parameter values for the motor 30 (unloaded) are given below.

Parameter	Value	Units
M	3	phases
N_p	4	pole pairs
J	0.3	$\frac{\text{g}\cdot\text{m}^2}{\text{rad}}$
L	diag{1.5, 1.5, 1.5}	mH
R_e	diag{14.3, 14.3, 14.3}	Ω
B	0.25	$\frac{\text{g}\cdot\text{m}^2}{\text{rad}\cdot\text{sec}}$
encoder	4000	counts/rev

The actual torque-angle characteristic functions, along with the actual cogging, are shown in Fig. 6. These plots were determined by measurements with a standard torque sensor. Note that the torque-angle characteristics are not close to sinusoidal, and as such, cannot be accurately approximated with a truncated Fourier series unless several terms are included. Thus, the motor chosen for the purpose of demonstrating the present invention highlights the advantages of the new piecewise-linear approximation.

Using 25 segments ($N_s = 25$), the assumed initial torque-angle characteristic functions are as shown in Fig. 7. Note that not only are the amplitude and "shape" of the initial estimates in error, but more importantly, the fundamental components of the assumed torque-angle curves are out of phase (by 0.225 rad) with their actual counterparts of Fig. 6. Such phase errors may be the consequence of a misalignment between the position sensor and the torque-angle characteristic functions. This means that the linearizing control will have large errors, resulting in very poor performance in the absence of adaptation (self-tuning).

It may seem that aligning the position sensor to the torque-angle curves is a simple procedure, and that this assumed lack of knowledge is unrealistic. For the motor presented here, which has only 4 pole pairs, this is perhaps true. However, for permanent-magnet synchronous motors used in position control applications (where N_p is far greater than for the chosen prototype motor), any misalignment between the encoder and the torque-angle characteristics is magnified. For instance, if the motor had 40 pole pairs, then a lack of knowledge of the phase of the torque-angle characteristic curves equal to that used here would occur if the

encoder and motor were misaligned by only about 0.02 rad. To achieve higher alignment accuracy than this for mass-production motors would require that either a custom factory setup be done for each motor, or that a possibly undesirable startup sequence be used. Note finally that this problem is completely neglected by adaptive schemes which use only a single term Fourier truncation, or if they simplify the Fourier series used by assuming symmetry and using only the $\sin(\cdot)$ terms.

A piecewise-linear parameterization could be used to approximate the cogging. From Fig. 6, however, it should be clear to those skilled in the art that the cogging of the prototype motor can be approximated with reasonable accuracy using

$$\bar{\eta}(\theta) := \begin{bmatrix} \sin(24\theta) & \cos(24\theta) \end{bmatrix} \begin{bmatrix} \eta_1^* \\ \eta_2^* \end{bmatrix} \quad (51)$$

Even though this parameterization requires computationally expensive transcendental function calls, the simplicity of the code which results is worth the cost. Of course, for a mass scale production, the more computationally efficient piecewise-linear parameterization would be used so that the least expensive microprocessor could be used. Rewriting the load torque parameterization of Equations 50 and 51 gives

$$\Theta_r^* = \begin{bmatrix} B \\ \eta_1^* \\ \eta_2^* \end{bmatrix} \quad (52)$$

$$w_r[n] = \begin{bmatrix} \omega[n] \\ \sin(24\theta[n]) \\ \cos(24\theta[n]) \end{bmatrix} \quad (53)$$

In all cases to follow, the identifiers were initialized with the values given below:

Parameter	Value	Units
$\hat{J}[0]$	1.0	$\frac{\text{g}\cdot\text{m}^2}{\text{rad}}$
$\hat{B}[0]$	1.0	$\frac{\text{g}\cdot\text{m}^2}{\text{rad}\cdot\text{sec}}$
$\hat{\eta}_1[0]$	0.02	$\text{N}\cdot\text{m}$
$\hat{\eta}_2[0]$	0.0	$\text{N}\cdot\text{m}$
$\hat{R}_{e_1}[0]$	15.0	Ω
$\hat{R}_{e_2}[0]$	15.0	Ω
$\hat{R}_{e_3}[0]$	15.0	Ω

Note that the cogging and resistances are initialized with rough estimates of their true values. This is done because it is reasonable to expect that some knowledge of these parameters will be available. The inertia and viscous friction terms, however, are initialized with values which have considerable error. This was done not only to emphasize the ability of the present invention to overcome such errors, but also because these parameters are more difficult to measure, and as such, their accurate knowledge is less likely.

The identifier gains used are given below:

Parameter	Value
γ_e	diag{0.025,0.005,0.005}
x_e	0.2
β_e	1.0
γ_m	diag{0.02,0.02,1.0,1.0}
x_m	0.01
β_m	0.0005

These values were determined by tuning the controller over several runs. It is emphasized, however, that the performance of the scheme is not unreasonably sensitive to these values, so one can expect performance similar to that described below using a wide range of gains.

To test the self-tuning velocity tracking method of the present invention, motor 30 was commanded to track a relatively difficult smooth trajectory, with a controller gain of $K_v = 0.7$. Adaptive tuning was enabled only after five seconds. The results are shown in Fig. 8A-C, with the Fig. 8A plot showing the desired and actual trajectories, the Fig. 8B plot showing the tracking error, and the Fig. 8C plot showing the instantaneous power supplied to the motor. From these plots, it is clear that the controller does a very poor job when adaptation is disabled, with tracking errors of as much as 36 rad/sec and power usage which is at or near the amplifier saturation limits. (Note that the power usage is asymmetric with respect to the sign of the velocity because of the initial phase error in the torque-angle characteristic functions.) When adaptation is enabled at $t = 5$ seconds, however, the response dramatically improves, with the tracking error decreasing to a steady state of about 0.25 rad/sec after only about 2 seconds of adaptation. The instantaneous power gives further evidence of the improvement resulting from the adaptive tuning of the present invention.

Similarly, dramatic results occur when using the method of the present invention for position trajectory tracking, with results shown in Fig. 9A-C. The gains used for this run were $K_p = -10$ and $K_v = 0.8$. As in the velocity case discussed above, the performance was very poor in the absence of adaptive tuning. The untuned tracking error is as much as 13 rad (about 2 revolutions), with instantaneous power again at or near amplifier saturation during the entire untuned portion of the run. After about 2.5 seconds of adaptive tuning, the tracking error is reduced to about 0.006 rad (about 4 encoder counts) during the constant position portions of the trajectory, and about 0.01 rad during the transitions. It is also evident from Fig. 9C that the tuning has reduced the power level to near that required for the motion.

While the present invention has been disclosed in preferred forms, it will be obvious to those skilled in the art that many modifications, additions, and deletions may be made therein without departing from the scope and spirit of the invention as set forth in the following claims.

Claims

1. A method for controlling the velocity or position of a permanent-magnet synchronous motor, the motor of the type having windings on the stator and permanent magnets on the rotor, to follow a desired position trajectory or a desired velocity trajectory, comprising the steps of:
 - (a) selecting initial electrical parameter estimates and initial mechanical parameter estimates of the motor;
 - (b) applying a voltage to the stator windings so as to command the motor to follow the desired position trajectory or desired velocity trajectory using the initial estimates of the electrical and mechanical parameters of the motor in a simplified mathematical model of the motor including a piecewise approximation for torque-angle characteristic functions of the motor;
 - (c) determining rotor position, rotor velocity, and currents in the stator windings;
 - (d) comparing the rotor position or the rotor velocity to the desired rotor position or the desired rotor velocity, respectively, so as to obtain a position error signal or a velocity error signal;
 - (e) computing updated electrical parameter estimates and updated mechanical parameter estimates of the motor;
 - (f) determining an updated simplified mathematical model of the motor using the updated electrical parameter estimates and mechanical parameter estimates of the motor;
 - (g) applying a voltage to the stator windings so as to command the motor to follow the desired position or desired velocity trajectory using the updated simplified mathematical model of the motor; and
 - (h) repeating steps (c) through (g).

2. The method as recited in Claim 1, wherein the simplified mathematical model of the motor includes a piecewise-linear approximation of the torque-angle characteristics of the motor.
3. The method as recited in Claim 1, wherein the simplified mathematical model of the motor includes a piecewise-polynomial approximation of the torque-angle characteristics of the motor.
4. The method as recited in Claim 1, wherein the simplified mathematical model of the motor further includes a piecewise approximation of the motor load.
5. The method as recited in Claim 1, wherein the step of determining rotor position, rotor velocity, and currents in the stator windings comprises determining the rotor velocity based upon numerical differentiation of a signal representing the rotor position.
6. The method as recited in Claim 1, wherein the windings on the stator have a stator phase self-inductance which is negligibly small.
7. The method as recited in Claim 1, wherein the simplified mathematical model of the motor comprises a discrete-time, reduced-order model based substantially completely on two mechanical states, the mechanical states being rotor position and rotor velocity.
8. The method as recited in Claim 1, wherein the step of computing updated electrical parameter estimates comprises using a previous amplifier input, the stator currents, the rotor position, and the rotor velocity to compute the electrical parameter estimates according to the following equation:

$$\hat{\Theta}_{e,j}[n+1] = \hat{\Theta}_{e,j}[n] + \frac{\gamma_e w_{e,j}[n]}{\kappa_e + w'_{e,j}[n] w_{e,j}[n]} G_{\beta_e}(y_{e,j}[n] - w'_{e,j}[n] \hat{\Theta}_{e,j}[n]).$$

9. The method as recited in Claim 1, wherein the step of computing updated mechanical parameter estimates comprises using a previous amplifier input, a present and previous rotor position measurement, a present and previous rotor velocity measurement, and the updated electrical parameter estimates to compute the updated mechanical parameter estimates according to the following equation:

$$\hat{\Theta}_m[n+1] = \hat{\Theta}_m[n] + \frac{\gamma_m w_m[n]}{\kappa_m + w'_m[n] w_m[n]} G_{\beta_m}(y_m(\hat{\Theta}_e[n], n-1) - w'_m[n] \hat{\Theta}_m[n]).$$

10. The method as recited in Claim 1, wherein the step of determining an updated simplified mathematical model of the motor further is carried out using the updated electrical parameter estimates and the updated mechanical parameter estimates.
11. A method for controlling the velocity or position of a permanent-magnet synchronous motor, the motor of the type having a stator with windings and a rotor with permanent magnets, the motor having associated therewith at least two electrical states, such as stator phase currents, and at least two mechanical states, such as rotor position and rotor velocity, comprising the step of:
 - (a) applying a voltage to the motor according to a reduced-order mathematical model of the motor based substantially exclusively on the mechanical states.
12. The method as claimed in Claim 11 further comprising the steps of:
 - (b) determining rotor position, rotor velocity, and currents in the stator windings;
 - (c) comparing the rotor position or the rotor velocity to the desired rotor position or the desired rotor velocity, respectively, so as to obtain a position error signal or a velocity error signal;

- (d) computing electrical parameter estimates and mechanical parameter estimates of the motor;
 - (e) determining an updated reduced-order mathematical model of the motor using the computed electrical parameter estimates and mechanical parameter estimates of the motor;
 - (f) applying a voltage to the stator windings so as to command the motor to follow the desired position or desired velocity trajectory using the updated reduced-order mathematical model of the motor; and
 - (g) repeating steps (b) through (f).
- 5
13. The method as claimed in Claim 11 wherein the reduced-order mathematical model of the motor includes a piecewise approximation for the torque-angle characteristic functions of the motor.
- 10
14. The method as recited in Claim 11, wherein the reduced-order mathematical model of the motor includes a piecewise-linear approximation of the torque-angle characteristics of the motor.
15. The method as recited in Claim 11, wherein the reduced-order mathematical model of the motor includes a piecewise-polynomial approximation of the torque-angle characteristics of the motor.
- 15
16. A method for controlling the position and/or velocity of a permanent-magnet synchronous motor, the motor of the type having a stator with windings and a rotor with permanent magnets, to follow a desired position trajectory and/or a desired velocity trajectory, comprising the steps of:
- 20
- (a) selecting initial parameter estimates of the characteristics of the motor;
 - (b) applying a voltage to the stator windings so as to command the motor to follow the desired position trajectory and/or the desired velocity trajectory using the initial parameter estimates of the characteristics of the motor in a reduced-order mathematical model of the motor including a piecewise-linear approximation for a torque-angle characteristic function of the motor;
 - (c) determining rotor position and/or rotor velocity;
 - (d) comparing the rotor position and/or the rotor velocity to the desired rotor position and/or the desired rotor velocity, respectively, so as to obtain a position error signal and/or a velocity error signal;
 - (e) Computing updated parameter estimates of the characteristics of the motor;
 - (f) determining an updated mathematical model of the motor using the position error signal and/or the velocity error signal and the updated parameter estimates of the characteristics of the motor;
 - (g) applying a voltage to the stator windings so as to command the motor to follow the desired position and/or velocity trajectory using the updated mathematical model of the motor; and
 - (h) repeating steps (c) through (g).
- 25
- 30
- 35
17. The method as recited in Claim 16, wherein the mathematical model of the motor comprises a discrete-time, reduced-order model in which the mechanical states of the motor evolve substantially slower than the electrical states of the motor.
- 40
18. The method as recited in Claim 16, wherein the reduced-order mathematical model of the motor is based substantially exclusively on mechanical states of the motor.
- 45
- 50
- 55

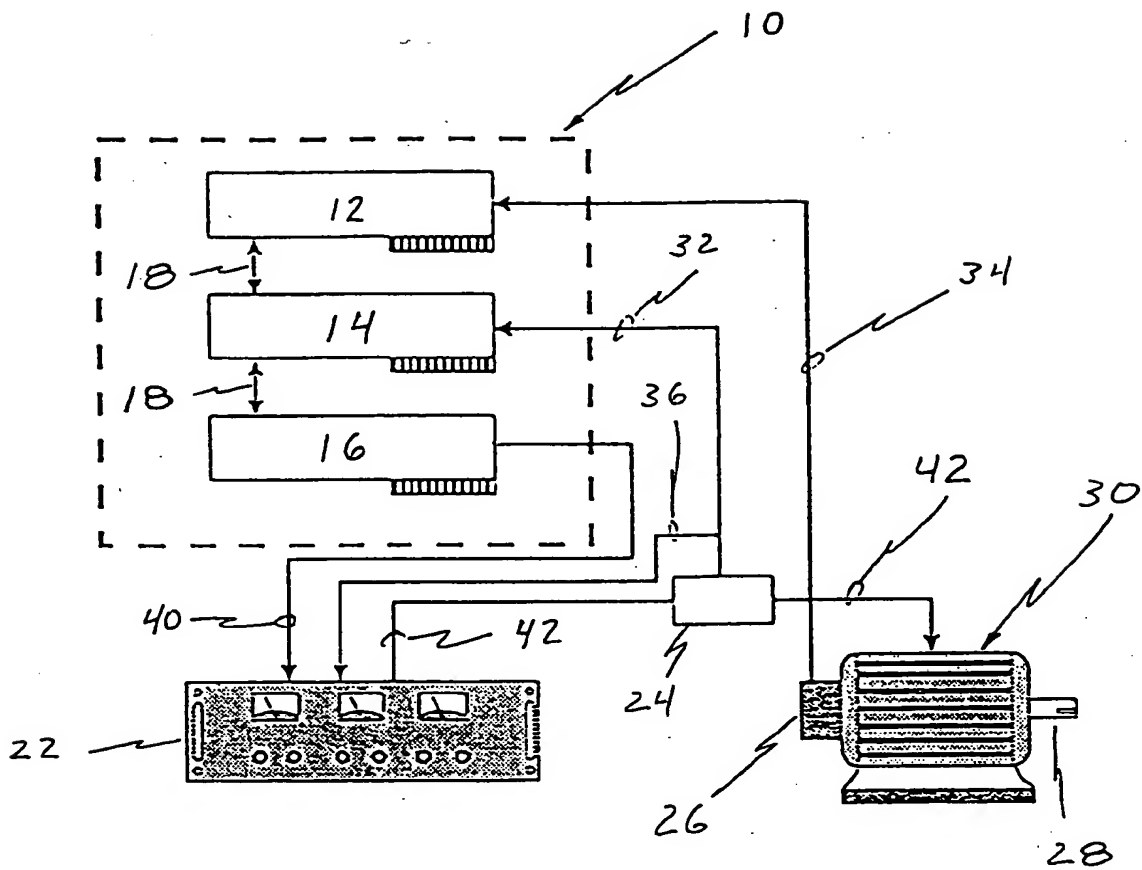


Fig. 1A

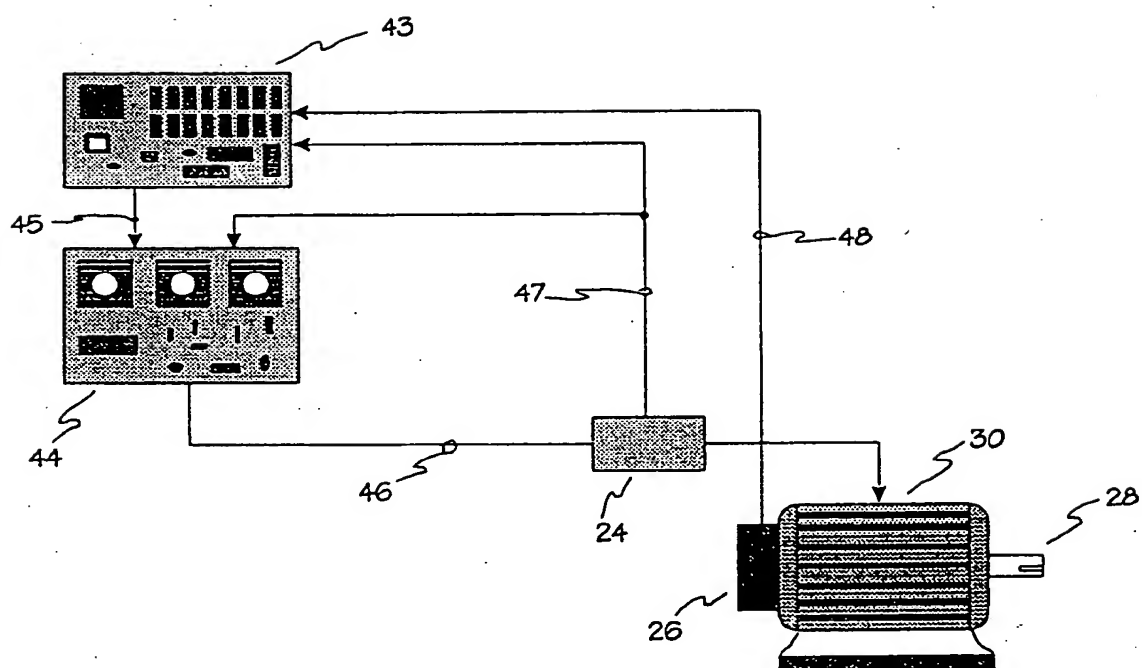


Fig. 1B

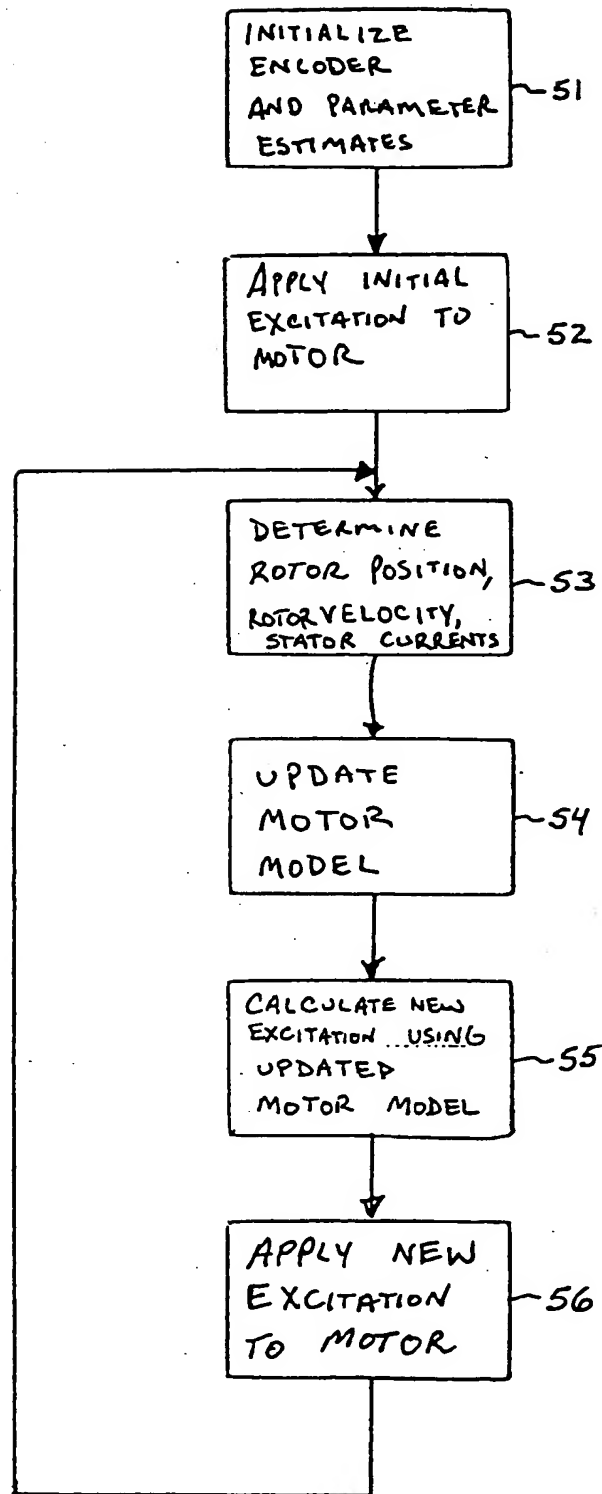


FIG 2A

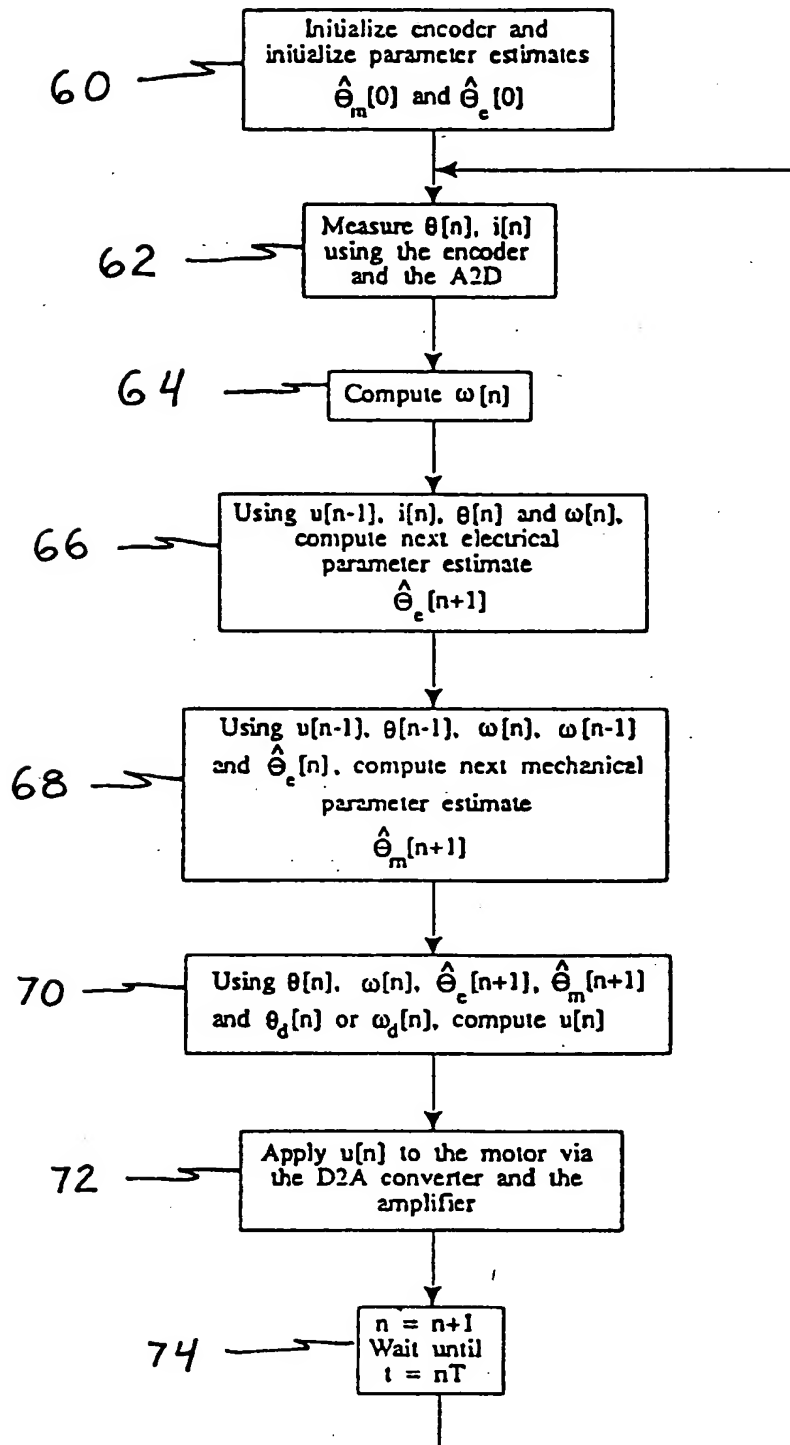


Fig. 2B

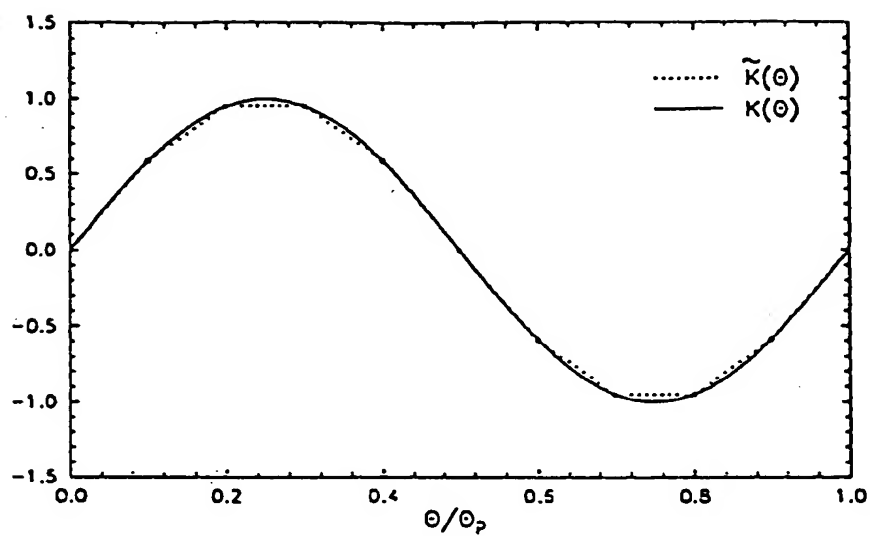


Fig. 3

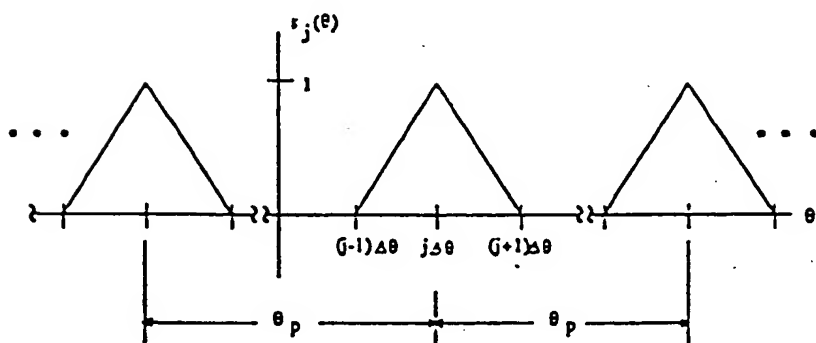


Fig. 4

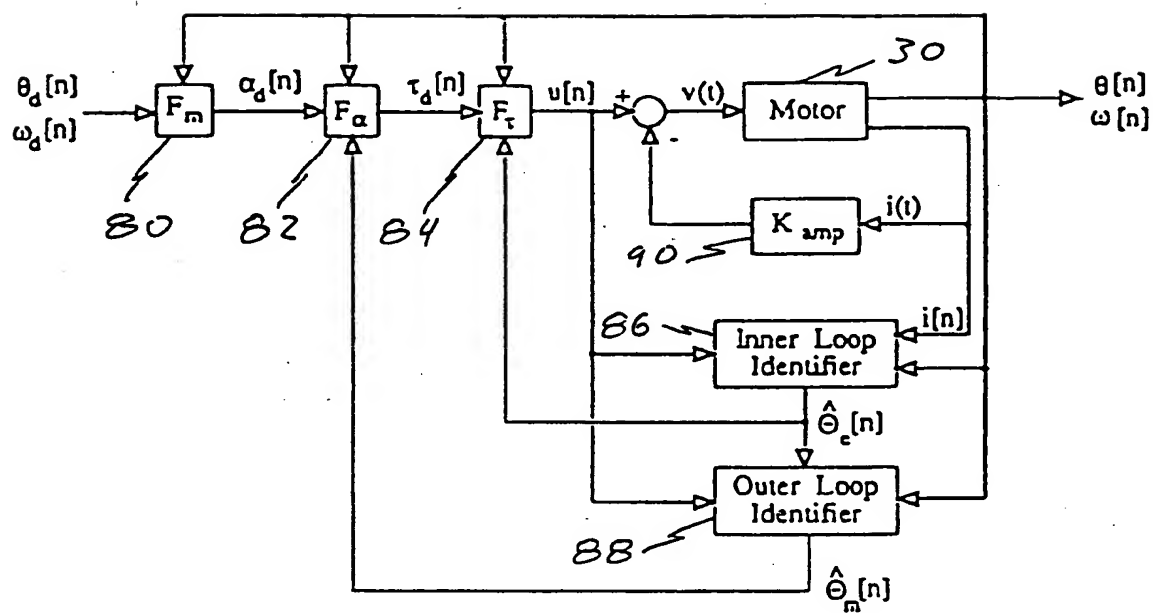


Fig. 5

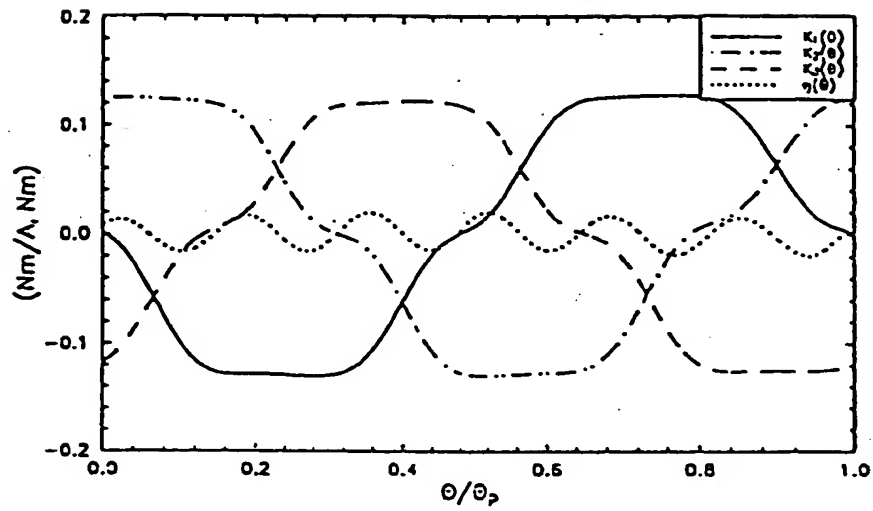


Fig. 6

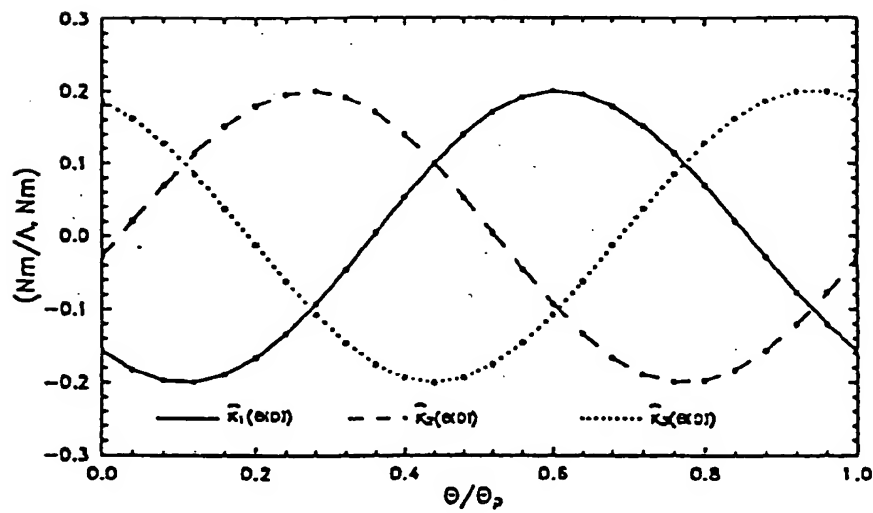


Fig. 7

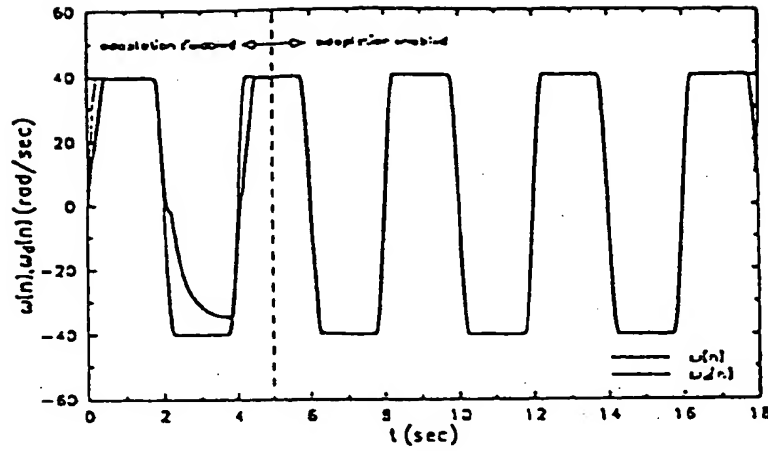


FIG 8A

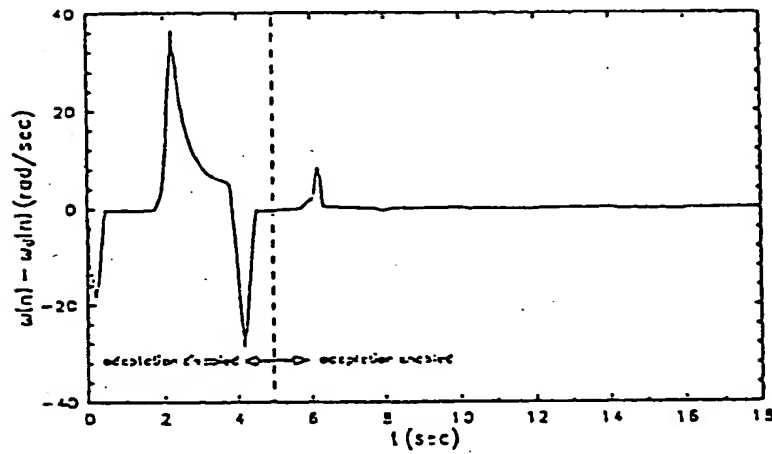


FIG 8B

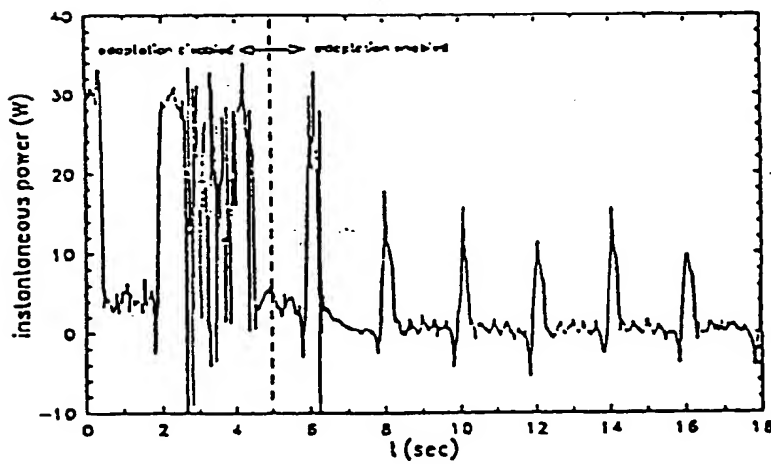


FIG 8C

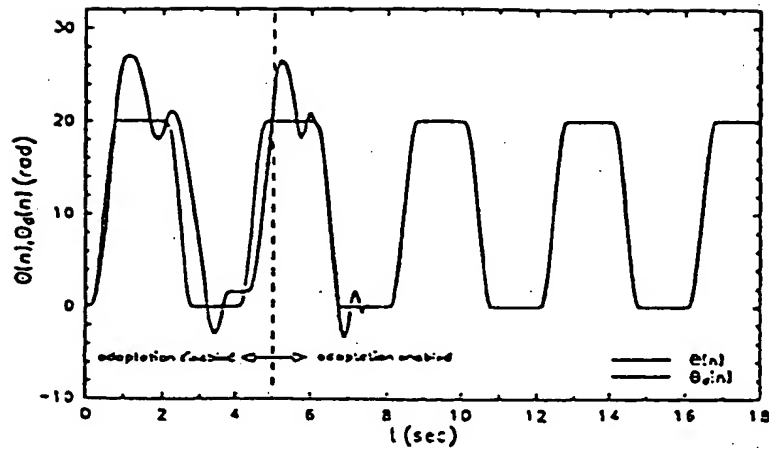


Fig 9A

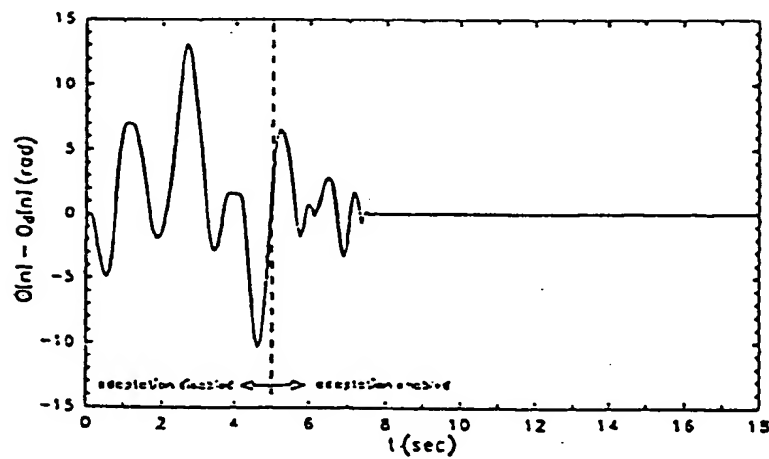


Fig 9B

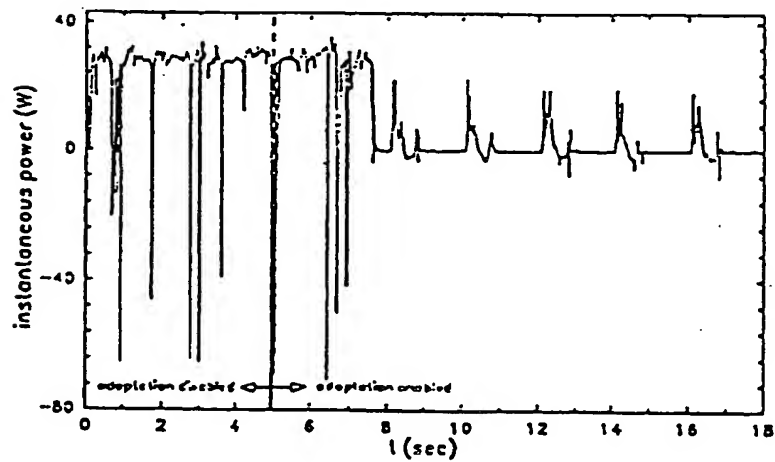


Fig 9C